The problem of absolute generality - 1

Øystein Linnebo

University of Oslo and Birkbeck, U. of London

28 May 2013
Is absolute generality possible?

Many of our quantifications are implicitly restricted.

(1) I have packed everything.

(2) The empty set has no members.

Some of our investigations and theories appear to require absolute generality, e.g. physicalism:

(3) Everything is physical.
Many of our quantifications are implicitly restricted.

(1) I have packed everything.
Many of our quantifications are implicitly restricted.

(1) I have packed everything.

However, some of our quantifications appear not to be thus restricted, for instance:

Øystein Linnebo (Oslo and London)
Is absolute generality possible?

Many of our quantifications are implicitly restricted.

(1) I have packed everything.

However, some of our quantifications appear not to be thus restricted, for instance:

(2) The empty set has no members.
Many of our quantifications are implicitly restricted.

(1) I have packed everything.

However, some of our quantifications appear not to be thus restricted, for instance:

(2) The empty set has no members.

Some of our investigations and theories appear to require absolute generality, e.g. physicalism:
Is absolute generality possible?

Many of our quantifications are implicitly restricted.

(1) I have packed everything.

However, some of our quantifications appear not to be thus restricted, for instance:

(2) The empty set has no members.

Some of our investigations and theories appear to require absolute generality, e.g. physicalism:

(3) Everything is physical.
But [the set-theoretic paradoxes] are only apparent 'contradictions', and depend solely on confusing set theory itself, which is not categorically determined by its axioms, with individual models representing it. What appears as an 'ultrafinite non- or super-set' in one model is, in the succeeding model, a perfectly good, valid set with both a cardinal number and an ordinal type, and is itself a foundation stone for the construction of a new domain. (Zermelo, 1930)
The argument from indefinite extensibility

There is a universal class $X$. But every class can, from an extended point of view, be regarded as a ‘perfectly good, valid set’. Since sets are well-founded, the set corresponding to $X$ cannot have been a member of $X$. So the universality of $X$ was merely relative, not absolute.
The argument from indefinite extensibility
There is a universal class $X$. But every class can, from an extended point of view, be regarded as a ‘perfectly good, valid set’. Since sets are well-founded, the set corresponding to $X$ cannot have been a member of $X$. So the universality of $X$ was merely relative, not absolute.

The semantic argument
Domains of quantification are sets. But standard set theory teaches us that there is no universal set. So no quantifier can range over absolutely everything, only over the elements of some set.
The main responses to the relativist arguments

Generality relativism: Improved versions of the arguments establish that it is indeed impossible to quantify over absolutely everything. (Parsons, 1974), (Parsons, 2006), (Glanzberg, 2004), (Fine, 2006); anticipated by (Russell, 1908)

Orthodox generality absolutism: It is possible to quantify over absolutely everything. And there is nothing special about this range of quantification. It is just as definite as that of, say, quantification over electrons. (Boolos, 1985), (Cartwright, 1994), (Williamson, 2003)

Alternative generality absolutism: It is possible to quantify over absolutely everything. But the totality of absolutely everything is, in a certain sense, indefinite. (Dummett, 1991), (Linnebo, 2006), (Linnebo, 2010); anticipated by Cantor
**Generality relativism**: Improved versions of the arguments establish that it is indeed impossible to quantify over absolutely everything. (Parsons, 1974), (Parsons, 2006), (Glanzberg, 2004), (Fine, 2006); anticipated by (Russell, 1908)
The main responses to the relativist arguments

- *Generality relativism*: Improved versions of the arguments establish that it is indeed impossible to quantify over absolutely everything. (Parsons, 1974), (Parsons, 2006), (Glanzberg, 2004), (Fine, 2006); anticipated by (Russell, 1908)

- *Orthodox generality absolutism*: It is possible to quantify over absolutely everything. And there is nothing special about this range of quantification. It is just as definite as that of, say, quantification over electrons. (Boolos, 1985), (Cartwright, 1994), (Williamson, 2003)

- *Alternative generality absolutism*: It is possible to quantify over absolutely everything. But the totality of absolutely everything is, in a certain sense, indefinite. (Dummett, 1991), (Linnebo, 2006), (Linnebo, 2010); anticipated by Cantor
The main responses to the relativist arguments

- **Generality relativism**: Improved versions of the arguments establish that it is indeed impossible to quantify over absolutely everything. (Parsons, 1974), (Parsons, 2006), (Glanzberg, 2004), (Fine, 2006); anticipated by (Russell, 1908)

- **Orthodox generality absolutism**: It is possible to quantify over absolutely everything. And there is nothing special about this range of quantification. It is just as definite as that of, say, quantification over electrons. (Boolos, 1985), (Cartwright, 1994), (Williamson, 2003)

- **Alternative generality absolutism**: It is possible to quantify over absolutely everything. But the totality of absolutely everything is, in a certain sense, *indefinite*. (Dummett, 1991), (Linnebo, 2006), (Linnebo, 2010); anticipated by Cantor
An investigation at the intersection of philosophy and logic

The motivation for the project and individual views examined is often quite philosophical. The execution of the project and its individual arguments rely heavily on logic and mathematics. Few, if any, philosophical views or arguments will be presupposed. All we presuppose is a willingness to reason outside of the (currently!) standard formal theories, such as ZFC. Such informal mathematical reasoning is legitimate and a necessary prerequisite to the formal reasoning. Most of all, we presuppose a willingness to think with an open mind about some hard logico-mathematical paradoxes.

Øystein Linnebo (Oslo and London)
The motivation for the project and individual views examined is often quite philosophical.
An investigation at the intersection of philosophy and logic

- The motivation for the project and individual views examined is often quite philosophical.
- The execution of the project and its individual arguments rely heavily on logic and mathematics.
The motivation for the project and individual views examined is often quite philosophical.

The execution of the project and its individual arguments rely heavily on logic and mathematics.

Few, if any, philosophical views or arguments will be presupposed.
The motivation for the project and individual views examined is often quite philosophical.

The execution of the project and its individual arguments rely heavily on logic and mathematics.

Few, if any, philosophical views or arguments will be presupposed.

All we presuppose is a willingness to reason outside of the (currently!) standard formal theories, such as ZFC. Such informal mathematical reasoning is legitimate and a necessary prerequisite to the formal reasoning.
The motivation for the project and individual views examined is often quite philosophical.

The execution of the project and its individual arguments rely heavily on logic and mathematics.

Few, if any, philosophical views or arguments will be presupposed.

All we presuppose is a willingness to reason outside of the (currently!) standard formal theories, such as ZFC. Such informal mathematical reasoning is legitimate and a necessary prerequisite to the formal reasoning.

Most of all, we presuppose a willingness to think with an open mind about some hard logico-mathematical paradoxes.
Plan for the four lectures

1. *Introduction*: the problem of absolute generality; higher-order logic; the paradoxes of indefinite extensibility
Plan for the four lectures

1. *Introduction*: the problem of absolute generality; higher-order logic; the paradoxes of indefinite extensibility

2. *Debating the extensibility claim*: responses to the paradox of indefinite extensibility
1. *Introduction:* the problem of absolute generality; higher-order logic; the paradoxes of indefinite extensibility

2. *Debating the extensibility claim:* responses to the paradox of indefinite extensibility

3. *Semantics and absolute generality:* the paradox of absolute generality; generality relativism dismissed; higher-order semantics
Plan for the four lectures

1. *Introduction*: the problem of absolute generality; higher-order logic; the paradoxes of indefinite extensibility

2. *Debating the extensibility claim*: responses to the paradox of indefinite extensibility

3. *Semantics and absolute generality*: the paradox of absolute generality; generality relativism dismissed; higher-order semantics

4. *Modal mathematics*: a modal explication of the notion of definiteness; modal set theory; property theory
Consider the claim that Socrates thinks, formalized as:

\[
\text{THINK}(\text{Socrates})
\] (1)
Consider the claim that Socrates thinks, formalized as:

\[ \text{THINK}(\text{Socrates}) \]  

(1)

First-order logic (FOL) allows us to generalize into the noun position to conclude:

\[ \exists x \text{ THINK}(x) \]  

(2)
Plural and higher-order logic (I)

Consider the claim that Socrates thinks, formalized as:

\[
\text{THINK}(\text{Socrates})
\]  

(1)

First-order logic (FOL) allows us to generalize into the noun position to conclude:

\[
\exists x \text{ THINK}(x)
\]  

(2)

Plural first-order logic (PFO) allows us to generalize \textit{plurally} into the noun position to conclude that there are one or more objects \(xx\) that think:

\[
\exists xx \text{ THINK}(xx)
\]  

(3)
Consider the claim that Socrates thinks, formalized as:

\[ \text{THINK}(\text{Socrates}) \]  \hspace{1cm} (1)

First-order logic (FOL) allows us to generalize into the noun position to conclude:

\[ \exists x \text{THINK}(x) \]  \hspace{1cm} (2)

Plural first-order logic (PFO) allows us to generalize \textit{plurally} into the noun position to conclude that there are one or more objects \(xx\) that think:

\[ \exists xx \text{THINK}(xx) \]  \hspace{1cm} (3)

Second-order logic (SOL) allows us to generalize into the predicate position to conclude that there is a concept \(F\) under which Socrates falls:

\[ \exists F F(\text{Socrates}) \]  \hspace{1cm} (4)
Further combinations and extensions may be possible as well:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SOL</td>
<td>PSO</td>
<td>...??</td>
</tr>
<tr>
<td>FOL</td>
<td>PFO</td>
<td>...??</td>
</tr>
</tbody>
</table>
The language

- Add predicate variables $F_i^n$
- Allow such variables to be bound by quantifiers
- Adding function variables is optional

Deductive systems for SOL

The usual I- and E-rules extended to the second-order quantifiers

Comprehension axioms which specify what values the second-order variables can take:

$$\exists F \forall x [Fx \leftrightarrow \phi(x)] \quad \text{(Comp)}$$

where $\phi(x)$ does not contain $F$ free.

A second-order choice axiom can be added if desired, but we won’t.
The language and logic of SOL

The language

- Add predicate variables $F^n_i$
- Allow such variables to be bound by quantifiers
- Adding function variables is optional

Deductive systems for SOL

- The usual I- and E-rules extended to the second-order quantifiers
- Comprehension axioms which specify what values the second-order variables can take:
  \[
  \exists F \forall x[Fx \leftrightarrow \phi(x)] \quad \text{(Comp)}
  \]
  where $\phi(x)$ does not contain $F$ free.
- A second-order choice axiom can be added if desired, but we won’t.
Plural logic

- Same language as that of monadic SOL, except that instead of $F_i^1 t$, we have $t \prec xx_i$ (read as ‘$t$ is one of $xx_i$’).

Same language as that of monadic SOL, except that instead of $F_i^1 t$, we have $t \prec xx_i$ (read as ‘$t$ is one of $xx_i$’).

‘$\exists xx$’ and ‘$\forall xx$’ read as ‘there are some objects $xx$ such that . . . ’ and ‘whenever there are some objects $xx$, . . . ’ respectively.
Plural logic

- Same language as that of monadic SOL, except that instead of $F_i^1 t$, we have $t \prec xx_i$ (read as ‘$t$ is one of $xx_i$').

- ‘$\exists xx$’ and ‘$\forall xx$’ read as ‘there are some objects $xx$ such that . . . ’ and ‘whenever there are some objects $xx$, . . . ’ respectively.

- Same deductive system, except adjustments required by there being no empty plurality:
Plural logic

- Same language as that of monadic SOL, except that instead of $F^1_i t$, we have $t \prec xx;_i$ (read as ‘$t$ is one of $xx;_i$’).

- ‘$\exists xx$’ and ‘$\forall xx$’ read as ‘there are some objects $xx$ such that . . . ’ and ‘whenever there are some objects $xx$, . . . ’ respectively.

- Same deductive system, except adjustments required by there being no empty plurality:
  - $\forall xx \exists u(u \prec xx)$
Plural logic

- Same language as that of monadic SOL, except that instead of $F^1_i t$, we have $t \prec xx_i$ (read as ‘$t$ is one of $xx_i$’).

- ‘$\exists xx$’ and ‘$\forall xx$’ read as ‘there are some objects $xx$ such that . . . ’ and ‘whenever there are some objects $xx$, . . . ’ respectively.

- Same deductive system, except adjustments required by there being no empty plurality:
  
  - $\forall xx \exists u (u \prec xx)$
  
  - $\exists u \phi(u) \rightarrow \exists xx \forall u [u \prec xx \leftrightarrow \phi(u)]$
Two further remarks

Pluralities and concepts are typically thought to have different modal properties:
Two further remarks

Pluralities and concepts are typically thought to have different *modal properties*:

\[
\exists F \exists x (Fx \land \Diamond \neg Fx)
\]
Pluralities and concepts are typically thought to have different modal properties:

- $\exists F \exists x (Fx \land \lozenge \neg Fx)$
- $\forall yy \forall x (x \prec yy \rightarrow \Box(Eyy \rightarrow (x \prec yy)))$
Two further remarks

Pluralities and concepts are typically thought to have different *modal properties*:

- $\exists F \exists x (Fx \land \lozenge \neg Fx)$
- $\forall yy \forall x (x \prec yy \rightarrow \Box (Eyy \rightarrow (x \prec yy)))$

**Ontological innocence**

- Plural logic is typically thought not to introduce any ontological commitments to sets, classes, reified ‘pluralities’, or the like.
Two further remarks

Pluralities and concepts are typically thought to have different modal properties:

- $\exists F \exists x (Fx \land \Diamond \neg Fx)$
- $\forall yy \forall x (x \prec yy \rightarrow \square (Eyy \rightarrow (x \prec yy)))$

Ontological innocence

- Plural logic is typically thought not to introduce any ontological commitments to sets, classes, reified ‘pluralities’, or the like.
- This will be particularly important when dealing with some of the problems surrounding absolute generality. For instance, ‘the sets’ refers to each and every set without commitment to any problematic universal set or class that mysteriously fails to be a set.
A *paradox* is an apparently unacceptable conclusion derived from apparently acceptable premises by means of apparently acceptable reasoning.

When confronted with a paradox, we cannot simply use *reductio ad absurdum* to reject one of the premises. Why is this the culprit rather than some other premise?

An *antinomy* is a particularly stubborn paradox where we are unable to identify any culprit. That is, we are unable to reject one of the premises or to accept the conclusion—at least given the concepts involved.
A *paradox* is an apparently unacceptable conclusion derived from apparently acceptable premises by means of apparently acceptable reasoning.

When confronted with a paradox, we cannot simply use *reductio ad absurdum* to reject one of the premises. Why is *this* the culprit rather than some other premise?
A *paradox* is an apparently unacceptable conclusion derived from apparently acceptable premises by means of apparently acceptable reasoning.

When confronted with a paradox, we cannot simply use *reductio ad absurdum* to reject one of the premises. Why is *this* the culprit rather than some other premise?

An *antinomy* is a particularly stubborn paradox where we are unable to identify any culprit. That is, we are unable to reject one of the premises or to accept the conclusion—at least given the concepts involved.
Let’s use ‘number’ in a completely general sense, much like our notion of ordinal number.
Let’s use ‘number’ in a completely general sense, much like our notion of ordinal number.

[Number totality]

It is permissible to talk about sequences of numbers, including a sequence of all numbers.
Let’s use ‘number’ in a completely general sense, much like our notion of ordinal number.

[Number totality]
It is permissible to talk about sequences of numbers, including a sequence of all numbers.

Why accept [Number totality]? 
- Mathematicians do talk about sequences and ‘collections’ of numbers, e.g. On
Let’s use ‘number’ in a completely general sense, much like our notion of ordinal number.

**[Number totality]**

It is permissible to talk about sequences of numbers, including a sequence of all numbers.

Why accept [Number totality]?

- Mathematicians do talk about sequences and ‘collections’ of numbers, e.g. On
- We can use plural or higher-order resources, e.g. talk about some numbers, etc.
[Number extensibility 1]
For any number \( \alpha \), there is a successor \( \alpha + 1 \)

Why accept [Number extensibility 1]?
We can always count one more
We can always add one item at the end of a sequence.

Why accept [Number extensibility 2]?
Cantor showed that the sequence of natural numbers can be given a LUB. Why not other sequences of numbers too? A differential treatment would have to be justified!

Merely to point to the threat of contradiction is 'to wield the big stick, not to offer an explanation' (Dummett, 1991, p. 316).

\( \text{Øystein Linnebo} \) (Oslo and London)

The problem of absolute generality - 1
[Number extensibility 1]
For any number $\alpha$, there is a successor $\alpha + 1$

[Number extensibility 2]
For any sequence of numbers, there is a least upper bound.
[Number extensibility 1]
*For any number* $\alpha$, *there is a successor* $\alpha + 1$

[Number extensibility 2]
*For any sequence of numbers, there is a least upper bound.*

*Why accept [Number extensibility 1]?*
- We can always count one more
- We can always add one item at the end of a sequence.

Cantor showed that the sequence of natural numbers can be given a LUB. Why not other sequences of numbers too? A differential treatment would have to be justified!

Merely to point to the threat of contradiction is 'to wield the big stick, not to offer an explanation' (Dummett, 1991, p. 316).
[Number extensibility 1]
For any number $\alpha$, there is a successor $\alpha + 1$

[Number extensibility 2]
For any sequence of numbers, there is a least upper bound.

Why accept [Number extensibility 1]?
- We can always count one more
- We can always add one item at the end of a sequence.

Why accept [Number extensibility 2]?
- Cantor showed that the sequence of natural numbers can be given a LUB. Why not other sequences of numbers too? A differential treatment would have to be justified!
[Number extensibility 1]
For any number $\alpha$, there is a successor $\alpha + 1$

[Number extensibility 2]
For any sequence of numbers, there is a least upper bound.

Why accept [Number extensibility 1]?
- We can always count one more
- We can always add one item at the end of a sequence.

Why accept [Number extensibility 2]?
- Cantor showed that the sequence of natural numbers can be given a LUB. Why not other sequences of numbers too? A differential treatment would have to be justified!
- Merely to point to the threat of contradiction is ‘to wield the big stick, not to offer an explanation’ (Dummett, 1991, p. 316).
The paradox of numbers (Burali-Forti) (II)

The two extensibility principles imply:
The two extensibility principles imply:

**[Number extensibility]**

*For every sequence of numbers, there is a larger number.*
The two extensibility principles imply:

[Number extensibility]
For every sequence of numbers, there is a larger number.

But clearly, [Number totality] and [Number extensibility] are inconsistent.
An analogous paradox arises for iterative sets (Linnebo, 2010).
An analogous paradox arises for iterative sets (Linnebo, 2010).

**[Set totality]**

*It is permissible to talk about ‘collections’ of sets (and other objects), including a collection of all sets.*
An analogous paradox arises for iterative sets (Linnebo, 2010).

[Set totality]

*It is permissible to talk about ‘collections’ of sets (and other objects), including a collection of all sets.*

[Set extensibility]

*For any ‘collection’ of sets, there is another set.*
The paradox of iterative sets (II)

[Set extensibility] follows from:

1. The membership relation $\in$ is well-founded.
2. Any 'collection' $X$ of objects corresponds to a set.

Why accept [Set extensibility 1]?

The members of a set are 'prior' to the set itself

(Oslo and London)

Øystein Linnebo
The paradox of iterative sets (II)

[Set extensibility] follows from:

[Set extensibility 1]
*The membership relation* $\in$ *is well-founded.*
The paradox of iterative sets (II)

[Set extensibility] follows from:

[Set extensibility 1]
The membership relation $\in$ is well-founded.

[Set extensibility 2]
Any ‘collection’ $X$ of objects corresponds to a set.
The paradox of iterative sets (II)

[Set extensibility] follows from:

[Set extensibility 1]
*The membership relation \( \in \) is well-founded.*

[Set extensibility 2]
*Any ‘collection’ \( X \) of objects corresponds to a set.*

*Why accept [Set extensibility 1]?*

- The members of a set are ‘prior’ to the set itself
[Set extensibility] follows from:

[Set extensibility 1]
*The membership relation* $\in$ *is well-founded.*

[Set extensibility 2]
*Any ‘collection’* $X$ *of objects corresponds to a set.*

*Why accept [Set extensibility 1]?
- The members of a set are ‘prior’ to the set itself
- The iterative conception of set (Boolos, 1971)
[Set extensibility 2]

Any ‘collection’ $X$ of objects corresponds to a set.
[Set extensibility 2]
Any ‘collection’ \(X\) of objects corresponds to a set.

Why accept [Set extensibility 2]?

- Traditionally, most mathematicians and philosophers denied the existence of completed infinities, including infinite sets. But as Cantor realized, some infinite collections do give rise to sets. Why, then, should not all collections do so? What is the principled difference between collections that do and do not give rise to sets?
[Set extensibility 2]
Any ‘collection’ $X$ of objects corresponds to a set.

Why accept [Set extensibility 2]?

- Traditionally, most mathematicians and philosophers denied the existence of completed infinities, including infinite sets. But as Cantor realized, some infinite collections do give rise to sets. Why, then, should not all collections do so? What is the principled difference between collections that do and do not give rise to sets?

- Part of the iterative conception: any ‘available’ objects can be used to form a set with precisely these objects as elements. And why shouldn’t any ‘collection’ be ‘available’?
How to respond to the paradoxes? (I)

The paradoxes of numbers and sets have a common structure (Russell, 1908).

1. Totality: There is a ‘collection’ of all Fs
2. Extensibility: Given any ‘collection’ of Fs, there is another F that is not in this ‘collection’.

We call paradoxes with this structure paradoxes of indefinite extensibility.

Recall that, when reasoning about a paradox, it is dialectically unacceptable simply to use reductio ad absurdum to reject one particular assumption. Why is this the culprit rather than some other assumption?
How to respond to the paradoxes? (I)

The paradoxes of numbers and sets have a common structure (Russell, 1908).

- **Totality**: There is a ‘collection’ of all $F$s
- **Extensibility**: Given any ‘collection’ of $F$s, there is another $F$ that is not in this ‘collection’.

Øystein Linnebo (Oslo and London)
How to respond to the paradoxes? (I)

The paradoxes of numbers and sets have a common structure (Russell, 1908).

- **Totality**: There is a ‘collection’ of all Fs
- **Extensibility**: Given any ‘collection’ of Fs, there is another F that is not in this ‘collection’.

We call paradoxes with this structure *paradoxes of indefinite extensibility*.
The paradoxes of numbers and sets have a common structure (Russell, 1908).

- **Totality**: There is a ‘collection’ of all $F$s
- **Extensibility**: Given any ‘collection’ of $F$s, there is another $F$ that is not in this ‘collection’.

We call paradoxes with this structure *paradoxes of indefinite extensibility*.

Recall that, when reasoning about a paradox, it is dialectically unacceptable simply to use *reductio ad absurdum* to reject one particular assumption. Why is *this* the culprit rather than some other assumption?
The main responses: denying one of the premises

Deny [Totality]?
This is unattractive, as we have plural and higher-order logic.

Deny [Extensibility]?
Not all totalities characterized by means of plural or higher-order logic correspond to objects.

This is probably the most widespread response today. It will be critically examined in lecture 2.
Deny [Totality]?

This is unattractive, as we have plural and higher-order logic.
The main responses: denying one of the premises

*Deny [Totality]?*

This is unattractive, as we have plural and higher-order logic.

*Deny [Extensibility]?*

Not all totalities characterized by means of plural or higher-order logic correspond to objects.

This is probably the most widespread response today. It will be critically examined in lecture 2.
According to generality relativism, the paradoxes result from a subtle context change.
According to generality relativism, the paradoxes result from a subtle context change.

The range of the quantifiers expands in the course of the argument because of a context change.

- There is a totality of $F$s in the old sense of the quantifiers.
- There is an $F$ outside of this totality in the new sense of the quantifiers.
According to generality relativism, the paradoxes result from a subtle context change.

The range of the quantifiers expands in the course of the argument because of a context change.

- There is a totality of *Fs in the old sense of the quantifiers*.
- There is an *F* outside of this totality *in the new sense of the quantifiers*.

This view does dispel the paradoxes. But the view is fraught with difficulties (Williamson, 2003), which we will seen in lecture 3.
Yet another possibility is that the paradoxes are due to an equivocation.
Yet another possibility is that the paradoxes are due to an equivocation.

By re-introducing intensional notions, we distinguish between definite and indefinite ‘collections’. We now see that the paradoxes trade on an equivocation.

<table>
<thead>
<tr>
<th></th>
<th>Totality</th>
<th>Extensibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>definite</td>
<td>✗</td>
<td>✓</td>
</tr>
<tr>
<td>indefinite</td>
<td>✓</td>
<td>✗</td>
</tr>
</tbody>
</table>
The main responses: appeal to modal notions

Yet another possibility is that the paradoxes are due to an equivocation.

By re-introducing intensional notions, we distinguish between definite and indefinite ‘collections’. We now see that the paradoxes trade on an equivocation.

<table>
<thead>
<tr>
<th></th>
<th>Totality</th>
<th>Extensibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>definite</td>
<td>✗</td>
<td>✓</td>
</tr>
<tr>
<td>indefinite</td>
<td>✓</td>
<td>✗</td>
</tr>
</tbody>
</table>

This response will be developed and examined in lecture 4.
The Iterative Conception of Set.
Reprinted in (Boolos, 1998).

Nominalist Platonism.
Reprinted in (Boolos, 1998).

*Logic, Logic, and Logic*.
Harvard University Press, Cambridge, MA.

Speaking of Everything.

*Frege: Philosophy of Mathematics*.
Harvard University Press, Cambridge, MA.

Oxford University Press, Oxford.


Relatively unrestricted quantification.


Quantification and Realism.


Sets, Properties, and Unrestricted Quantification.

Linnebo, Ø. (2010).

Pluralities and Sets.
Sets and classes.
*Noûs*, 8:1–12.
Reprinted in (Parsons, 1983).

*Mathematics in Philosophy*.
Cornell University Press, Ithaca, NY.

The problem of absolute universality.

Russell, B. (1908).
Mathematical logic as based on a theory of types.

Everything.
Zermelo, E. (1930).
Über Grenzzahlen und Mengenbereiche.
*Fundamenta Mathematicae*, 16:29–47.
Translated in (Ewald, 1996).